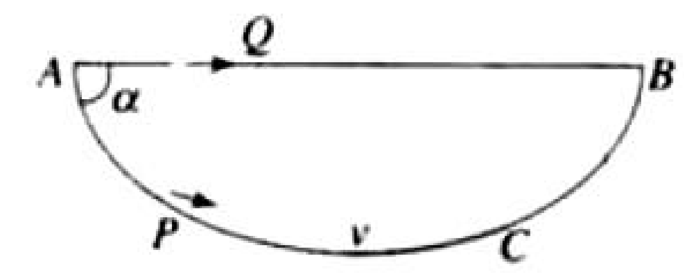
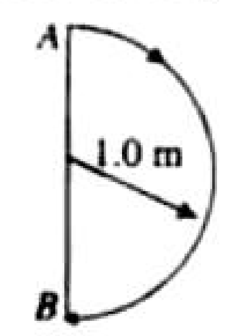
JEE Advanced

Single Correct Answer Type

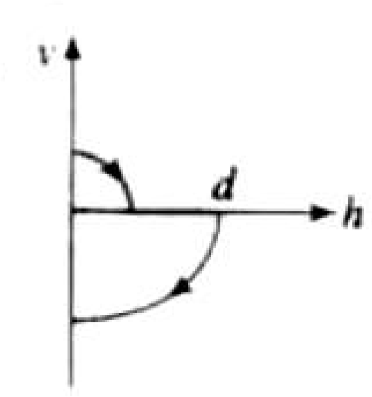
1. A ship of mass 3×10^7 kg initially at rest is pulled by a force of 5×10^4 N through a distance due to water is negligible, the speed of the ship.
   a. 1.5 m/s b. 60 m/s
   c. 0.1 m/s d. 5 m/s **(IIT-JEE 1980)**
2. A particle is moving eastwards with a velocity of 5 m s^{-1}. In 10 s, the velocity changes to 5 m s^{-1} northwards. The average acceleration in this time is
   a. Zero
   b. $1/\sqrt{2}$ m s^{-2} towards north-west
   c. 1/2 m s^{-2} towards north-west
   d. 1/2 m s^{-2} towards north **(IIT-JEE 1982)**
3. A river is flowing from west to east at a speed of 5 m per min. A man on the south bank of the river, capable of swimming at 10 m per min in still water, wants to swim across the river in the shortest time. He should swim in a direction
   a. Due north b. 30° east of north
   c. 30° west of north d. 60° east of north
   (IIT-JEE 1983)
4. A boat which has a speed of 5 km h^{-1} in still water crosses a river of width 1 km along the shortest possible path in 15 min. The velocity of the river water in kmh^{-1} is
   a. 1 b. 3
   c. 4 d. $\sqrt{41}$ **(IIT-JEE 1988)**
5. A ball is projected vertically upwards with a certain initial speed. Another ball of the same mass is projected at an angle of 60° with the vertical with the same initial speed. At the highest point, the ratio of their potential energies will be
   a. 4 : 1 b. 3 : 2 c. 2 : 3 d. 2 : 1
   (IIT-JEE 1989)
6. A body moving in a circular path with a constant speed has a
   a. constant velocity
   b. constant momentum
   c. constant kinetic energy
   d. constant acceleration **(IIT-JEE 1992)**
7. A particle moving in a straight line covers half the distance with a speed of 3 m/s. The other half of the distance is covered in two equal time intervals with speed of 4.5 m/s and 7.5 m/s respectively. The average speed of the particle during this motion is
   a. 4.0 m/s b. 5.0 m/s
   c. 5.5 m/s d. 4.8 m/s **(IIT-JEE 1992)**
8. A particle P is sliding down a frictionless hemispherical bowl. It passes the point A at $t = 0$. At this instant of time, the horizontal component of its velocity is v. A bead Q of the same mass as P is ejected from A at $t = 0$ along the horizontal string AB, with a speed v. Friction between the bead and the string may be neglected. Let t_P and t_Q be the respective times taken by P and Q to reach the point B. Then

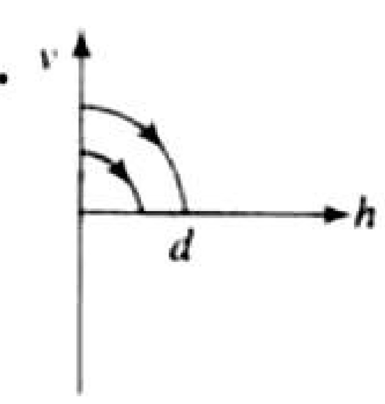


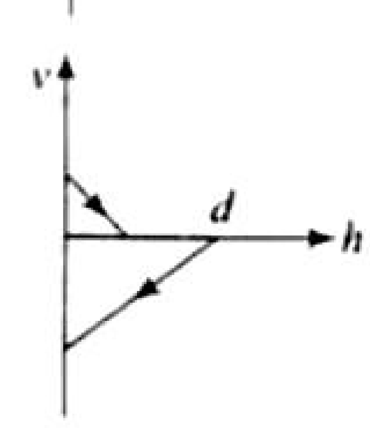
   a. $t_P < t_Q$ b. $t_P = t_Q$
   c. $t_P > t_Q$ d. $\dfrac{t_P}{t_Q} = \dfrac{\text{length of arc } ACB}{\text{length of chord } AB}$
   (IIT-JEE 1993)
9. A particle initially (i.e., at $t = 0$) moving with a velocity u is subjected to a retarding force, as a result of which it decelerates at a rate $a = -k\sqrt{v}$ where v is the instantaneous velocity and k is a positive constant. The time T taken by the particle to come to rest is given by
   a. $T = \dfrac{2\sqrt{u}}{k}$ b. $T = \dfrac{2u}{k}$
   c. $T = \dfrac{2u^{3/2}}{k}$ d. $T = \dfrac{2u^2}{k}$
   (IIT-JEE 1995)
10. In 1.0 s, a particle goes from point A to point B, moving in a semicircle of radius 1.0 m (see figure). The magnitude of the average velocity is

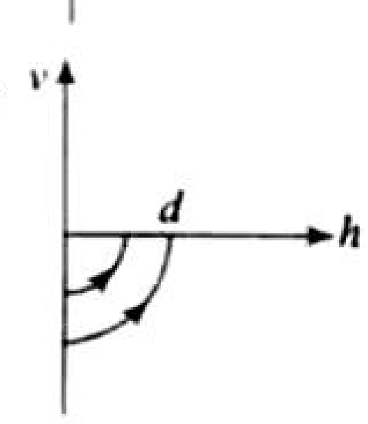


   a. 3.14 m s^{-1} b. 2.0 m s^{-1}
   c. 1.0 m s^{-1} d. Zero **(IIT-JEE 1999)**
11. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground is

   a.


   b.


   c.


   d.


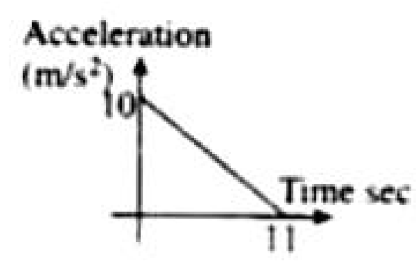
   (IIT-JEE 2000)
12. A particle starts sliding down a frictionless inclined plane. If S_n is the distance travelled by it from time $t = n - 1$ second to $t = n$ second, the ratio S_n/S_{n+1} is

a. $\frac{2n-1}{2n+1}$ b. $\frac{2n+1}{2n}$

c. $\frac{2n}{2n+1}$ d. $\frac{2n+1}{2n-1}$

(IIT-JEE 2004)

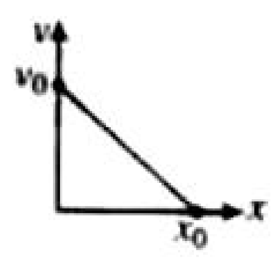
13. A particle starts from rest. Its acceleration (a) versus time (t) is as shown in the figure. The maximum speed of the particle will be:



a. 110 ms^{-1} b. 55 ms^{-1}
c. 550 ms^{-1} d. 660 ms^{-1}

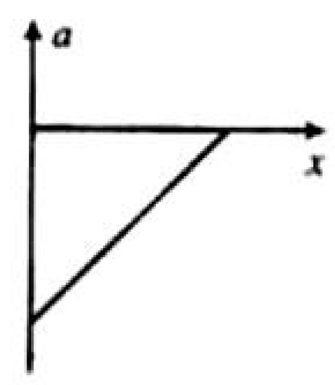
(IIT-JEE 2004)

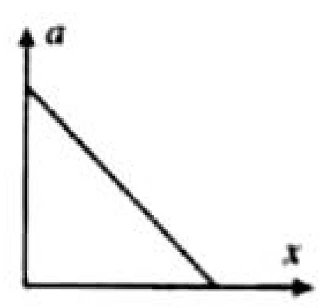
14. The velocity–displacement graph of a particle moving along a straight line is shown in the figure.

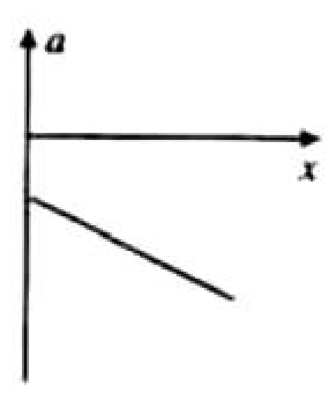


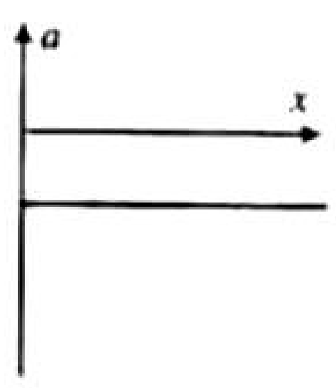
The most suitable acceleration–displacement graph will be

(IIT-JEE 2005)

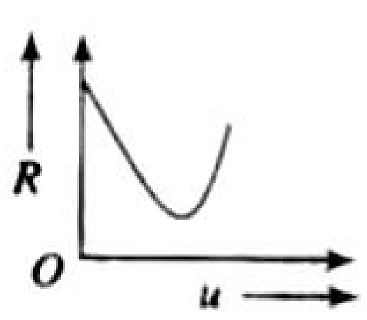
a. 

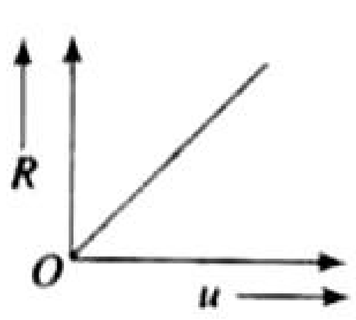
b. 

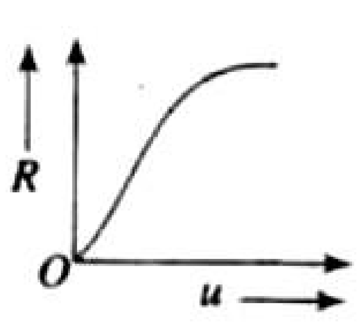
c. 

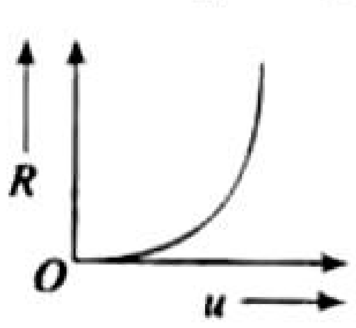
d. 

15. A projectile is projected with a velocity u at an angle θ with the horizontal. For a fixed θ, which of the graphs shown in the following figure shows the variation of range R versus u?

a. 

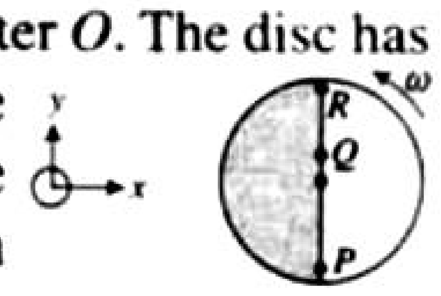
b. 

c. 

d. 

(IIT-JEE 2008)

16. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its center O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection in the y–z plane is the same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed 1/8 rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then



a. P lands in the shaded region and Q in the unshaded region.
b. P lands in the unshaded region and Q in the shaded region.
c. Both P and Q land in the unshaded region.
d. Both P and Q land in the shaded region.

(IIT-JEE 2012)

Multiple Correct Answer Type

1. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that

a. Velocity is constant
b. Acceleration is constant
c. Kinetic energy is constant
d. It moves in a circular path **(IIT-JEE 1987)**

2. The displacement x of a particle varies with time according to the relation $x = \frac{a}{b}(1 - e^{-bt})$. Then

a. At $t = 1/b$, the displacement of the particle is nearly $(2/3)(a/b)$.
b. The velocity and acceleration of the particle at $t = 0$ are a and $-ab$ respectively.
c. The particle cannot reach a point at a distance x' from its starting position if $x' > a/b$.
d. The particle will come back to its starting point as $t \to \infty$. **(IIT-JEE 1990)**

3. A particle of mass m moved on the x-axis as follows: it starts from rest $t = 0$ from the point $x = 0$, and comes to rest at $t = 1$ at the point $x = 1$. No other information is available about its motion at intermediate times ($0 < t < 1$). If α denotes the instantaneous acceleration of the particle, then

a. α cannot remain positive for all t in the interval $0 \le t \le 1$.
b. $|\alpha|$ cannot exceed 2 at any point in its path.
c. $|\alpha|$ must be ≥ 4 at some point or points in its path.
d. α must change sign during the motion, but no other assertion can be made with the information given.

(IIT-JEE 1993)

4. The coordinates of a particle moving in a plane are given by $x(t) = a\cos(pt)$ and $y(t) = b\sin(pt)$, where a, $b(< a)$, and p are positive constants of appropriate dimensions. Then

a. The path of the particle is an ellipse.
b. The velocity and acceleration of the particle are normal to each other at $t = \pi/2p$.

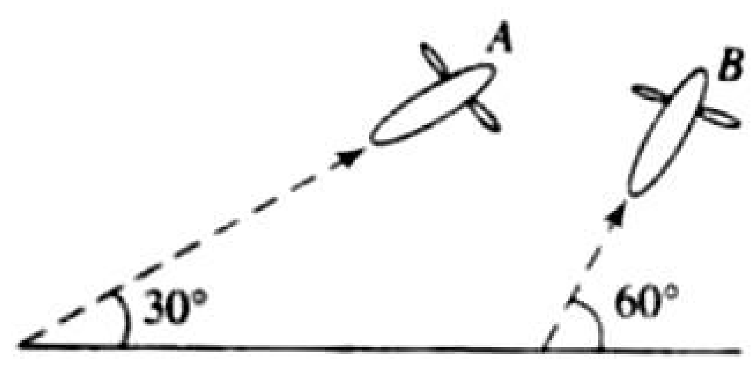
c. The acceleration of the particle is always directed towards a focus.

d. The distance travelled by the particle in time interval $t = 0$ to $t = \pi/2p$ is a. **(IIT-JEE 1999)**

Integer Answer Type

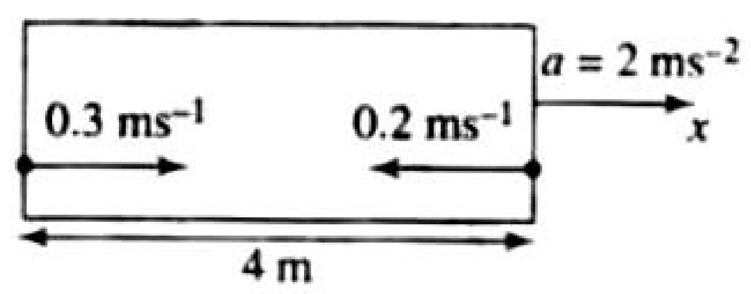
1. A train is moving along a straight line with a constant acceleration a. A boy standing in the train throws a ball forward with a speed of 10 m s^{-1}, at an angle of 60° to the horizontal. The body has to move forward by 1.15 m inside the train to catch the ball back to the initial height. The acceleration of the train, in m s^{-2}, is **(IIT-JEE 2011)**

2. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in the figure. The speed of A is $100\sqrt{3}$ ms^{-1}. At time $t = 0$ s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is



(JEE Advanced 2014)

3. A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along $+x$ direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in $+x$ direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in $-x$ direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is



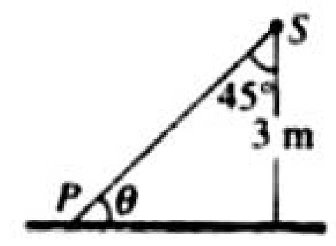
(JEE Advanced 2014)

Fill in the Blanks Type

1. A particle moves in a circle of radius R. In half the period of revolution its displacement is _______and distance covered is_______. **(IIT-JEE 1983)**

2. Four persons K, L, M, N are initially at the four corners of a square of side d. Each person now moves with a uniform speed v in such a way that K always moves directly towards L, L directly towards M, M directly towards N, and N directly towards K. The four persons will meet at a time______. **(IIT-JEE 1984)**

3. Spotlight S rotates in a horizontal plane with constant angular velocity of 0.1 rad s^{-1}. The spot of light P moves along the wall at a distance of 3 m. The velocity of the spot P when $\theta = 45°$ is______ m s^{-1}. **(IIT-JEE 1987)**



4. The trajectory of a projectile in a vertical plane is $y = ax - bx^2$, where a, b are constants, and x and y are, respectively, the horizontal and vertical distances of the projectile from the point of projection. The maximum height attained is ______ and the angle of projection from the horizontal is______. **(IIT-JEE 1997)**

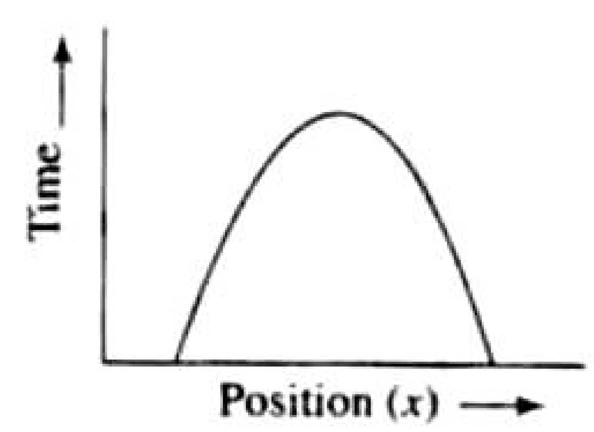
True/False Type

1. Two balls of different masses are thrown vertically upwards with the same speed. They pass through the point of projection in their downward motion with the same speed (neglect air resistance). **(IIT-JEE 1983)**

2. A projectile fired from the ground follows a parabolic path. The speed of the projectile is minimum at the top of its path. **(IIT-JEE 1984)**

Subject Type

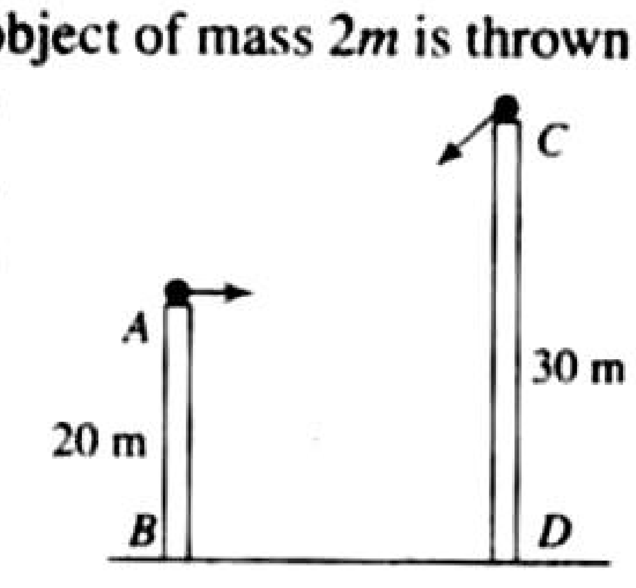
1. Answer the following giving reasons in brief: If the time variation of position, shown in the figure observed in nature?



(IIT-JEE 1979)

2. A body falling freely from a given height 'H' hits an inclined plane in its path at a height 'h'. As a result of this impact the direction of the velocity of the body becomes horizontal. For what value of (h/H) the body will take maximum time to reach the ground? **(IIT-JEE 1986)**

3. Two towers AB and CD are situated at distance d apart as shown in the figure. AB is 20 m high and CD is 30 m high from the ground. An object of mass m is thrown from the top of AB horizontally with a velocity of 10 m/s towards CD. Simultaneously another object of mass $2m$ is thrown from the top of CD at an angle 60° to the horizontal towards AB with the same magnitude of initial velocity as that of the first object. The two objects move in the same vertical plane, collide in mid-air and stick to each other.

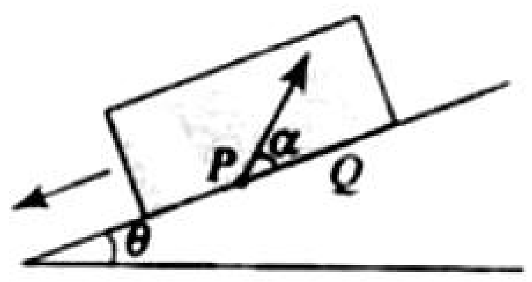


(i) Calculate the distance d between the towers.

(ii) Find the position where the objects hit the ground. **(IIT-JEE 1994)**

4. Two guns, situated on the top of a hill of height 10 m, fire one shot each with the same speed $5\sqrt{3}$ ms^{-1} at some interval of time. One gun fires horizontally and other fires upward at an angle of 60° with the horizontal. The shots collide in air at a point P. Find (i) the time-interval between the fringes and (ii) the coordinates of the point P. Take origin of the coordinate system at the foot of the hill right below the muzzle and trajectories in x-y plane. **(IIT-JEE 1996)**

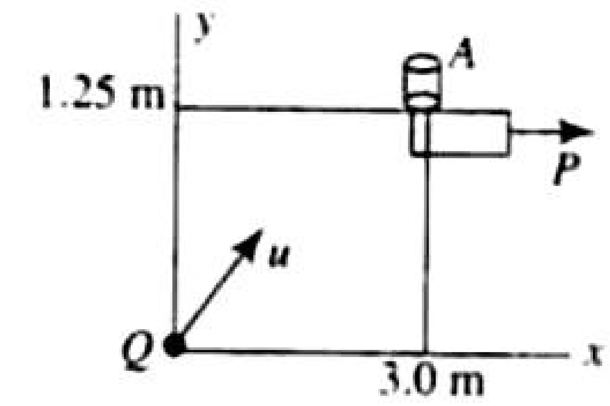
5. A large, heavy box is sliding without friction down a smooth plane of inclination θ. From a point P on the bottom of the box, a particle is projected inside the box. The initial speed of the particle with respect to the box is u, and the direction of projection makes an angle α with the bottom as shown in the figure.



(i) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance).

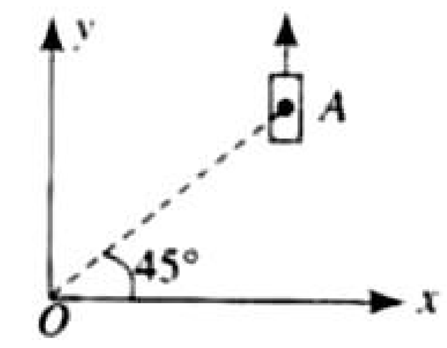
(ii) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when particle was projected. **(IIT-JEE 1998)**

6. An object A is kept fixed at the point x = 3 m and y = 1.25 m on a plank P raised above the ground. At time t = 0 the plank starts moving along the $+x$ direction with an acceleration 1.5 m/s^2. At the same instant a stone is projected from the origin with a velocity u as shown. A stationary person on the ground observes the stone hitting the object during its downward motion at an angle of 45° to the horizontal. All the motions are in the x-y plane. Find $\vec{u}$ and the time after which the stone hits the object.



(IIT-JEE 2000)

7. On a frictionless horizontal surface, assumed to be the x-y plane, a small trolley A is moving along a straight line parallel to the y-axis (see figure) with a constant velocity of $(\sqrt{3}-1)$ m/s. At a particular instant, when the line OA makes an angle of 45° with the x-axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle ϕ with the x-axis and it hits the trolley.



a. The motion of the ball is observed from the frame of the trolley. Calculate the angle θ made by the velocity vector of the ball with the x-axis in this frame.

b. Find the speed of the ball with respect to the surface, if $\phi = 4\theta/3$. **(IIT-JEE 2002)**

ANSWER KEY

JEE Advanced

Single Correct Answer Type

1. c.	**2.** b.	**3.** a.	**4.** b.	**5.** a.
6. c.	**7.** d.	**8.** a.	**9.** a.	**10.** b.
11. a.	**12.** a.	**13.** b.	**14.** a.	**15.** d.
16. c.				

Multiple Correct Answers Type

1. c, d. **2.** a, b, c. **3.** a, d.
4. a, b.

Integer Answer Type

1. (5) **2.** (5) **3.** (2)

Fill in the Blanks Type

1. $2r, \pi r$ **2.** d/v **3.** 0.6 **4.** $\frac{a^2}{4b}$

True/False Type

1. True. **2.** True.

Subjective Type

1. No **2.** $\frac{1}{2}$ **3.** (i) 17.32 m; (ii) 11.547 m

4. $(5\sqrt{3}\,\text{m}, 5m)$

5. (i) $\dfrac{2u\sin\alpha}{g\cos\theta}$ (ii) $\dfrac{u\cos(\alpha+\theta)}{\cos\theta}$ (down the plane)

6. $(3.75\hat{i} + 6.25)\hat{j}$ m/s

7. a. 45° b. 2 ms^{-1}

HINTS AND SOLUTIONS

JEE Advanced

Single Correct Answer Type

1. c. $v^2 = 2as = 2\left(\frac{F}{m}\right)s$ [As $u = 0$]

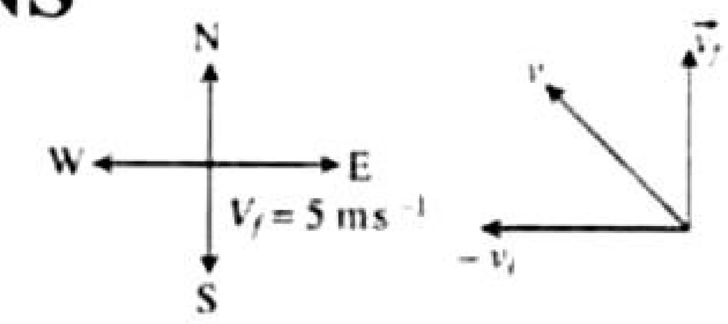
$$\Rightarrow \quad v^2 = 2\left(\frac{5\times 10^4}{3\times 10^7}\right)\times 3 = \frac{1}{100}$$

$$\Rightarrow \quad v = 0.1 \text{ m/s}$$

2. b. Average acceleration

$$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{t} = \frac{\vec{v}_f + (-\vec{v}_i)}{t} = \frac{\vec{v}}{t}$$

To find the resultant of $\vec{v}_f$ and $-\vec{v}_i$, we draw the following figure.



$$|\vec{v}| = \sqrt{v_f^2 + v_i^2} = \sqrt{5^2 + 5^2} = 5\sqrt{2} \text{ ms}^{-1}$$

Since, $|\vec{v}_f| = |-\vec{v}_i|$

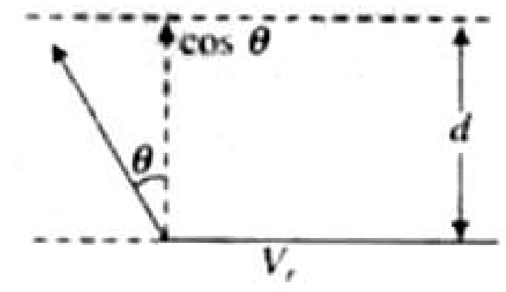
$\vec{v}$ is directed in between $\vec{v}_f$ and $-\vec{v}_i$.

Therefore, $\vec{v}$ is directed towards N-W.

$$\vec{a}_{av} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}$$

3. a. Time taken to cross the river $t = \frac{d}{v_s \cos\theta}$

For time to be minimum, $\cos\theta = \max \Rightarrow \theta = 0°$



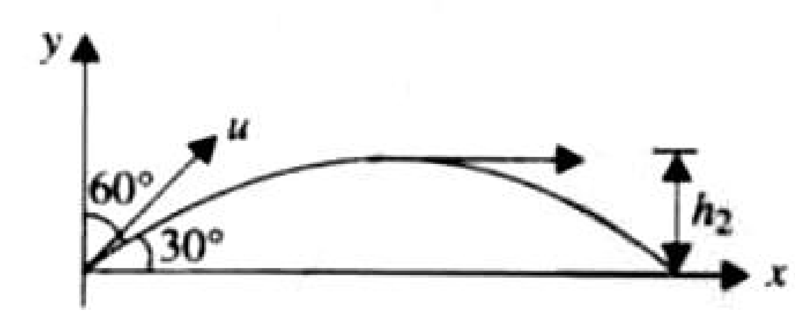
Hence, the swimmer should swim due north.

4. b. Net velocity of boat in river $= \sqrt{5^2 - u^2}$

$$t = \frac{\text{Distance}}{\text{Velocity}} \Rightarrow \frac{1}{4} = \frac{1}{\sqrt{5^2 - u^2}} \Rightarrow u = 3 \text{ km h}^{-1}$$

5. a. The maximum height attained by the first ball is $h_1 = \frac{u^2}{2g}$ where u is the initial speed of projection.
The maximum height attained by the second ball is

$$h_2 = \frac{u^2\sin^2(30°)}{2g} = \frac{u^2}{8g} \qquad (\because \theta = 90° - 60° = 30°)$$



Now, potential energy of the first ball at height $h_1 = mgh_1$ and that of the second ball at height $h_2 = mgh_2$. Therefore, the ratio of potential energies $\frac{mgh_1}{mgh_2} = \frac{h_1}{h_2} = \frac{u^2}{2g} \times \frac{8g}{u^2} = 4$

6. c. The direction of the velocity changes from point to point on the circle, choices (a), (b) and (d) are incorrect as velocity, momentum and acceleration are vector quantities. The speed is constant, hence kinetic energy will be constant.

7. d. Let the total distance be S. The time taken to travel the first half, i.e., $\frac{S}{2}$ is $t_1 = \frac{S/2}{3} = \frac{S}{6}$

Let t_2 be the time taken to cover a distance S_1 with speed 4.5 m/s and t_3 that cover distance S_2 with speed 7.5 m/s.
Then $S_1 = 4.5\, t_2$ and $S_2 = 7.5\, t_3$

Now $S_1 + S_2 = \frac{S}{2}$ and $t_2 = t_3$ (given)

Therefore $\frac{S}{2} = S_1 + S_2 = (4.5 + 7.5)t_2$ or $t_2 = \frac{S}{24}$.

$\therefore$ Total time taken $= t_1 + t_2 = \frac{S}{6} + \frac{S}{24} = \frac{5S}{24}$

$\therefore$ Average speed $= \frac{\text{total distance}}{\text{total time}} = \frac{S}{5S/24}$

$= \frac{24}{5} = 4.8$ m/s

8. a. If we consider the motion of the particle P along curved path ACB, motion between AC will be an accelerated one while between CB a retarded one. But in both case horizontal component of its velocity will be greater than or equal to v. Now considering the motion of particle P. In this case the velocity of particle Q is always equal to v. Horizontal displacements for both the particles are equal. Therefore, $t_P < t_Q$.

9. a. Given $a = -k\sqrt{v}$ or $\frac{dv}{dt} = -k\sqrt{v}$

Thus $v^{-1/2}\, dx = -k\, dt$

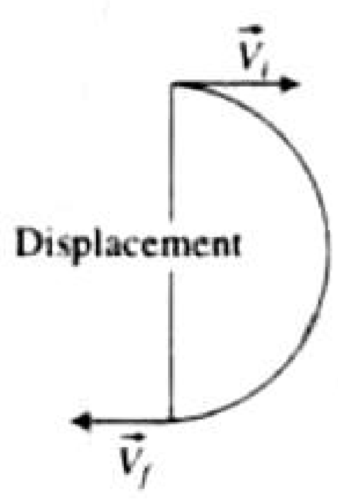
Integrating, we get $2v^{1/2} = -kt + c$. Using the given initial condition ($v = u$ at $t = 0$), we get $v = 2\sqrt{u}$. Thus we have

$$2(v^{1/2} - u^{1/2}) = -kt$$

Now, use $t = T$ and $v = 0$.

10. b. $|\text{Average velocity}| = \frac{|\text{Displacement}|}{\text{Time}}$

$= \frac{2r}{t} = 2 \times \frac{1}{1} = 2\ \text{m s}^{-1}$



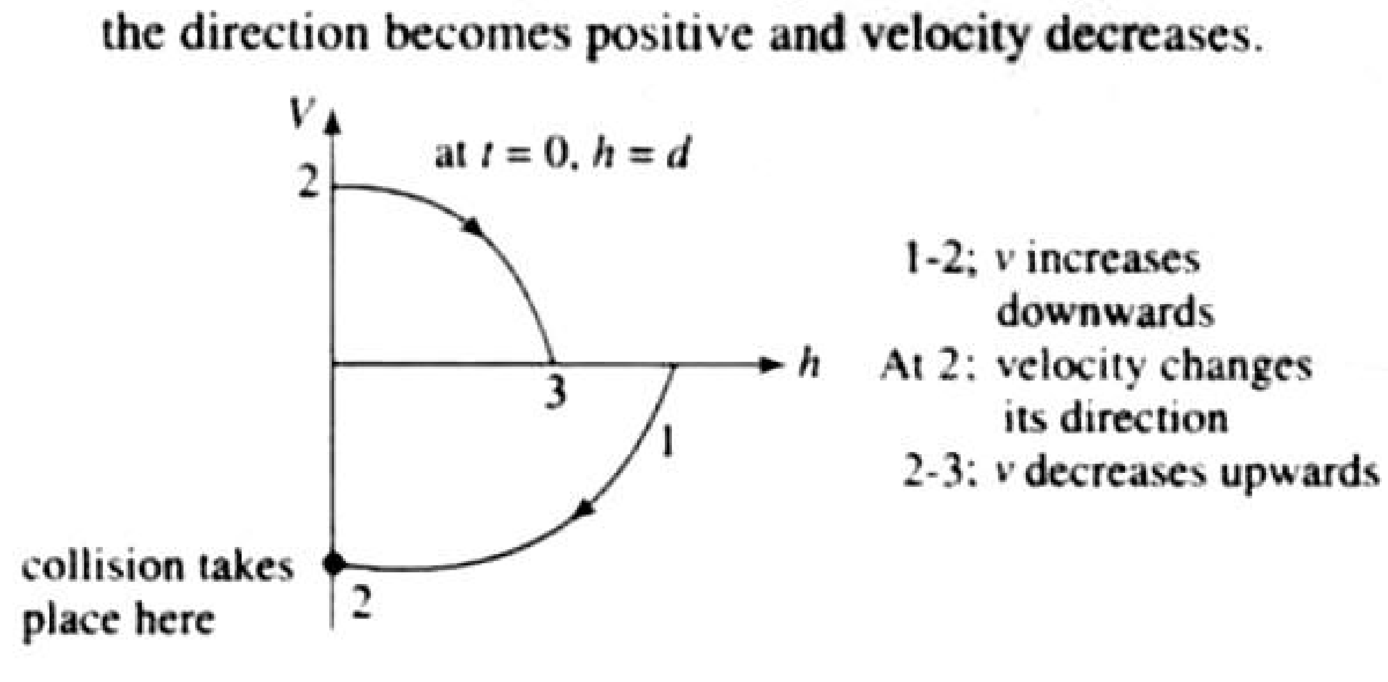
11. a. For the given condition initial height $h = d$ and velocity of the ball is zero. When the ball moves downward its velocity increases and it will be maximum when the ball hits the ground and just after the collision it becomes half and in opposite direction. As the ball moves upward its velocity again decreases and becomes zero at height $d/2$. This explanation matches with graph (a).

Alternate solution

We know for uniformly accelerated/decelerated motion $v^2 = u^2 \pm 2gh$

Before hitting the ground, the velocity v is given by $v^2 = 2gh$ (quadratic equation and hence parabolic path)

Downwards direction means negative velocity. After collision, the direction becomes positive and velocity decreases.



As the direction is reversed and speed is decreased and hence graph (a) represents these conditions correctly.

12. a. $S_n = \frac{a}{2}(2n - 1)$; $\quad S_{n+1} = \frac{a}{2}(2n + 1)$

$$\frac{S_n}{S_{n+1}} = \frac{2n - 1}{2n + 1}$$

13. b. Till 11 s, acceleration is positive, so velocity will go on increasing up to 11 s and maximum velocity will happen at 11 s.

The area under the acceleration–time graph gives change in velocity. Since particle starts with $u = 0$, change in velocity is

$V_f - V_i = V_{max} - 0 =$ Area under a–t graph

or $\quad v_{max} = \frac{1}{2} \times 10 \times 11 = 55\ \text{m s}^{-1}$

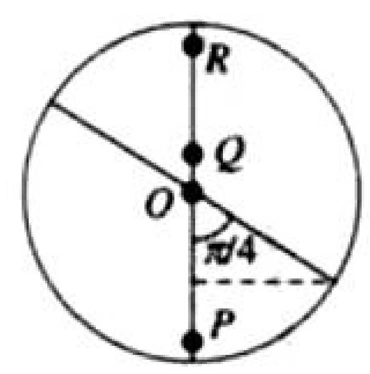
14. a. $v = \frac{v_0}{x_0}x + v_0$, $\quad a = v\frac{dx}{dt} = \left(-\frac{v_0}{x_0}x + v_0\right)\left(\frac{-v_0}{x_0}\right)$

$$\Rightarrow \quad a = \left(\frac{v_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$$

15. d. We have $R = \frac{u^2 \sin 2\theta}{g}$

For a fixed θ, $R \propto u^2$. So the correct graph is (d).

16. c. At $t = \frac{1}{8} \times \frac{2\pi}{\omega} = \frac{\pi}{4\omega}$



x-coordinate of $P = \omega R\left(\frac{\pi}{4\omega}\right) = \frac{\pi R}{4} > R\cos 45°$

Therefore, both particles P and Q land in unshaded region.

Multiple Correct Answer Type

1. c, d.

In the given condition, the particle undergoes uniform circular motion and for uniform circular motion the velocity and acceleration vector changes continuously but kinetic energy is constant for every point.

2. a, b, c.

Velocity of the particle is given by

$$v = \frac{dx}{dt} = \frac{d}{dt}\left\{\frac{a}{b}(1 - e^{-bt})\right\} = ae^{-bt}$$

Acceleration of the particle is given by

$$\alpha = \frac{dv}{dt} = \frac{d}{dt}(ae^{-bt}) = -abe^{-bt}$$

At $t = 1/b$, the displacement of the particle is

$$x = \frac{a}{b}(1 - e^{-1}) = \frac{a}{b}\left(1 - \frac{1}{3}\right) = \frac{2}{3}\frac{a}{b}$$

$$\left(\because e^{-1} = \frac{1}{3}\right)$$

Hence choice (a) is correct. At $t = 0$, the value v and α respectively are $v = ae^{-0} = a$ and $\alpha = -abe^{-0} = -ab$. Hence choice (b) is also correct. The displacement x is maximum when $t \to \infty$,

i.e., $x_{max} = \frac{a}{b}(1 - e^{-\infty}) = \frac{a}{b}$. Hence choice (c) is also correct.

Thus the correct choices are (a), (b) and (c).

3. a, d.

α cannot remain positive for all t in the interval $0 \le t \le 1$. This is because since the body starts from rest, it will first accelerate, finally it stops therefore a will become negative. Therefore, a will change its direction. Hence, (a) and (d) are the correct options.

Alternate solution

Since the body is at rest at $x = 0$ and $x = 1$. Hence, α cannot be positive for all time in the interval $0 \le t \le 1$. Therefore, first the particle is accelerated and then retarded. Now, total time $t = 1$s (given)

Total displacement $s = 1$m (given)

s = area under v–t graph

Height or $v_{max} = \frac{2s}{t} = 2$ m/s is also fixed.

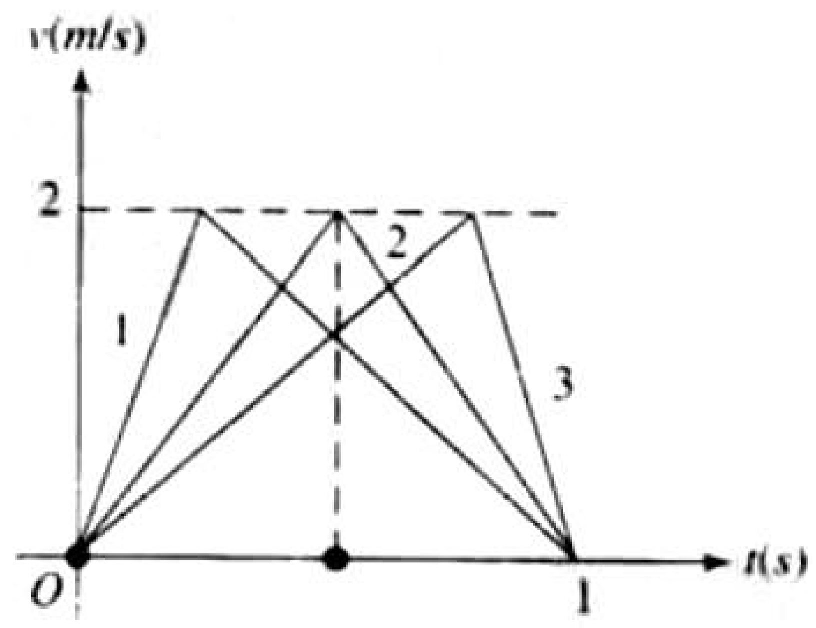
$$\left[\text{Area or } s = \frac{1}{2} \times t \times v_{max}\right]$$

If height and base are fixed, area is also fixed

In case 2: Acceleration = Retardation = 4 m/s^2

In case 1: Acceleration > 4 m/s^2 while

Retardation < 4 m/s^2



While in case 3: Acceleration < 4 m/s^{-2} and Retardation > 4 m/s^2

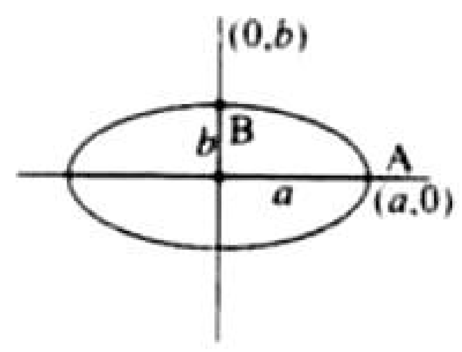
Hence, $|\alpha| \ge 4$ at some point or points in its path.

4. a, b.

$x = a \cos (pt)$, $y = b \sin (pt)$

Equation of path in x–y plane is

$$\left[\frac{x}{a}\right]^2 + \left[\frac{y}{b}\right]^2 = 1$$



i.e., the path of the particle is an ellipse.

Position vector of a point P is

$$\vec{r} = a \cos pt\,\hat{i} + b \sin pt\,\hat{j}$$

$$\Rightarrow \quad \vec{v} = p(-a \sin pt\,\hat{i} + b \cos pt\,\hat{j})$$

and $\vec{a} = -p^2(-a \cos pt\,\hat{i} + b \sin pt\,\hat{j}) = -p^2\vec{r}$

Acceleration directed towards the centre.

Also, $\vec{v} \cdot \vec{a} = 0$ at $t = \pi/2p$.

At $t = 0$, $\vec{r} = a\hat{i}$

At $t = \pi/2p$, $\vec{r} = b\hat{j}$

So in time $t = 0$ to $t = \pi/2p$, particle goes from A to B travelling a distance more than a.

Integer Type

1.(5) $T = \frac{2u \sin\theta}{g} = \frac{2 \times 10 \times \sqrt{3}}{10 \times 2} = \sqrt{3}$ s

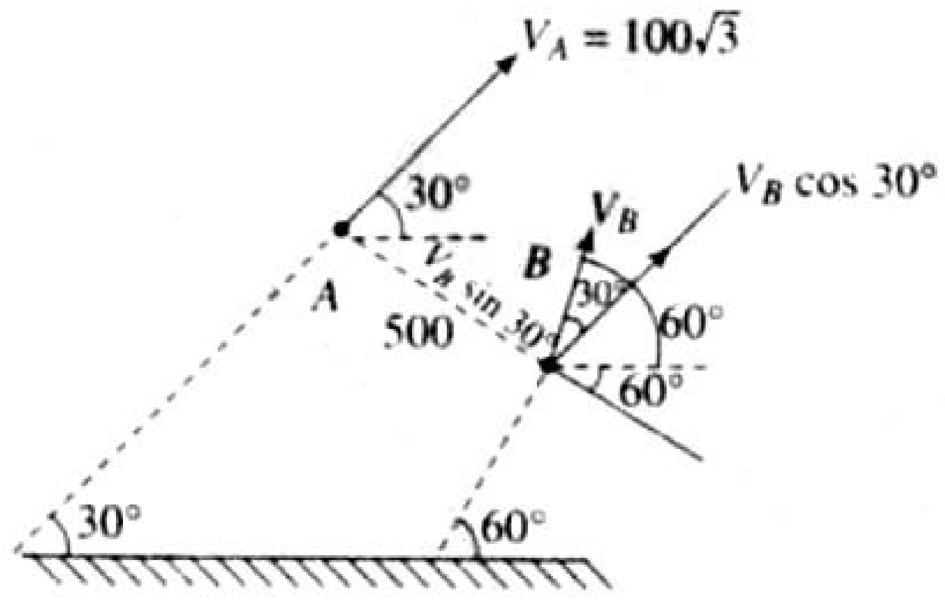
$$R = u \cos\theta\, T - \frac{1}{2} aT^2$$

$$1.15 = 10 \times \frac{1}{2}\sqrt{3} - \frac{1}{2}a(\sqrt{3})^2$$

$$\frac{3}{2}a = 5\sqrt{3} - 1.15 = 8.65 - 1.15 = 7.5$$

$$a = 7.5 \times \frac{2}{3} = 5 \text{ ms}^{-2}$$

2. (5) For relative motion perpendicular to line of motion of A

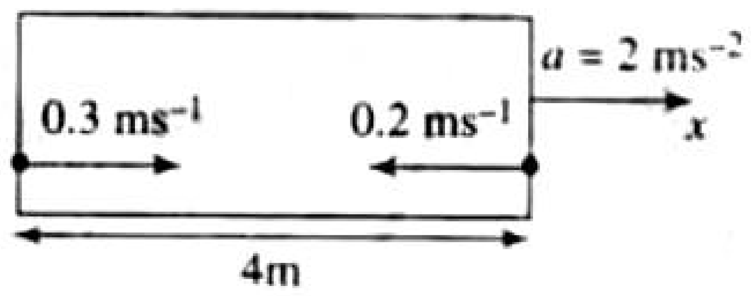


$$V_A = 100\sqrt{3} = V_B \cos 30°$$

$$\Rightarrow \quad V_B = 200 \text{ m/s}$$

$$t_0 = \frac{500}{V_B \sin 30°} = \frac{500}{200 \times \frac{1}{2}} = 5 \text{ sec}$$

3. (2) Consider motion of two balls with respect to rocket



Maximum distance of ball A from left wall

$$= \frac{u^2}{2a} = \frac{0.3 \times 0.3}{2 \times 2} = \frac{0.09}{4} \approx 0.02 \text{ m}$$

So collision of two balls will take place very near to left wall

For $\quad BS = ut + \frac{1}{2}at^2$

$$-4 = -0.2t - \left(\frac{1}{2}\right)2t^2 \Rightarrow t^2 + 0.2t - 4 = 0$$

$$\Rightarrow \quad t = \frac{-0.2 \pm \sqrt{0.04 + 16}}{2} = 1.9$$

nearest integer = 2 s

Fill in the Blanks Type

1. Displacement = $2r$
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Hence choice (a) is correct. At $t = 0$, the value v and α respectively are $v = ae^{-0} = a$ and $\alpha = -abe^{-0} = -ab$. Hence choice (b) is also correct. The displacement x is maximum when $t \to \infty$,

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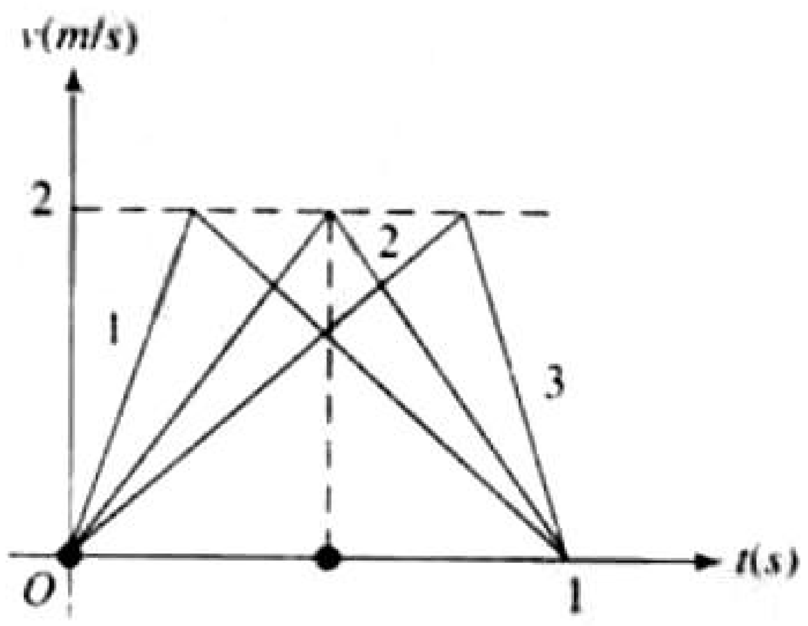

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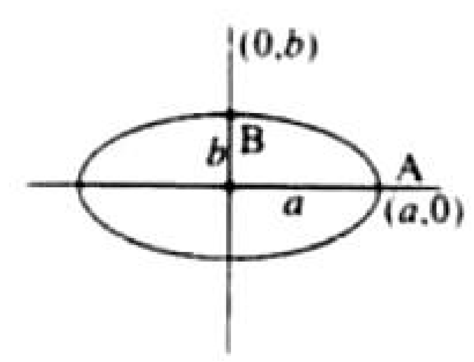

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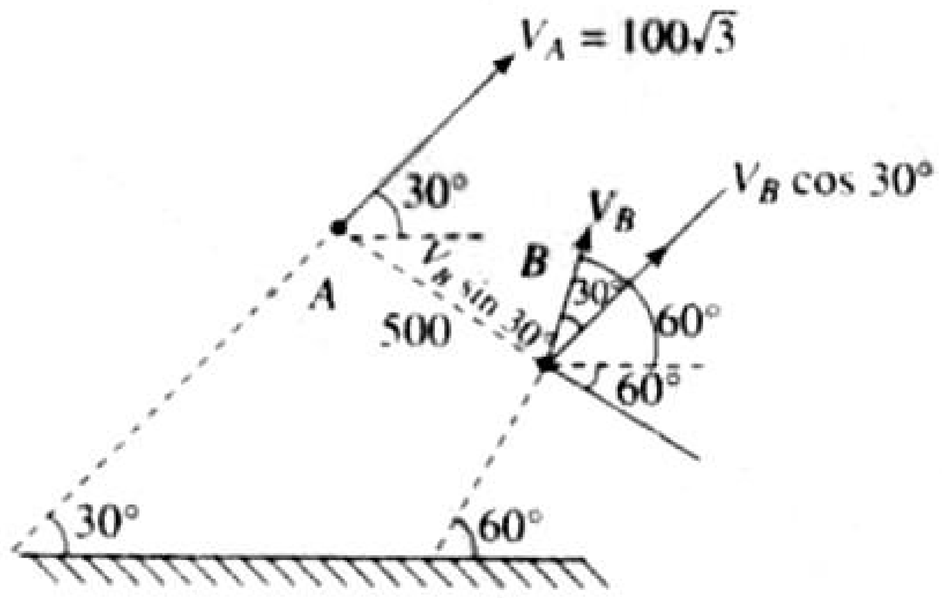

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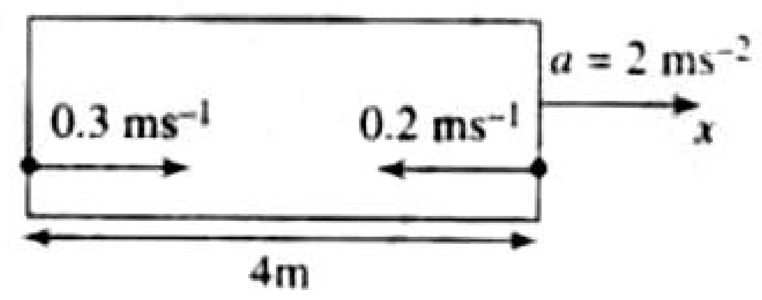

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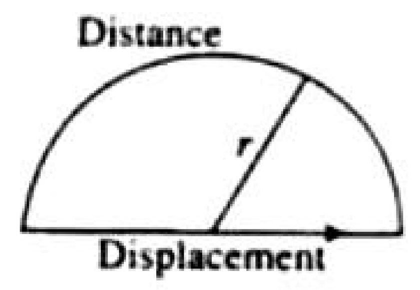
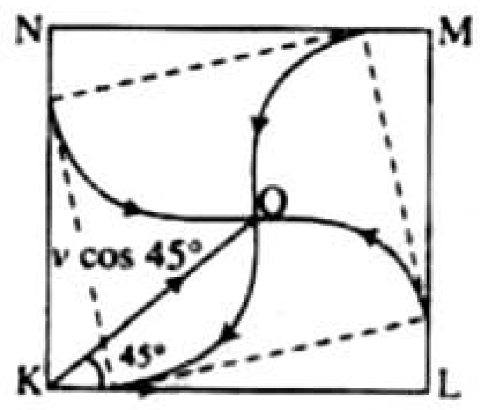


2. The velocity of K throughout the motion towards the centre of the square is $v \cos 45°$ and the displacement covered by this velocity will be KO.

$$t = \frac{KO}{v \cos 45°} = \frac{(\sqrt{2}d/2)}{v/\sqrt{2}} = \frac{d}{v}$$



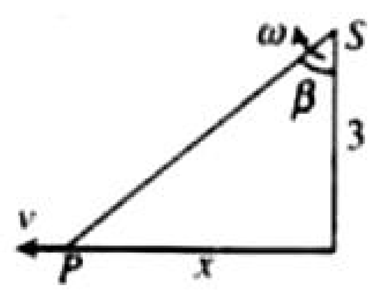
Alternatively:

$$\vec{v}_{KN} = \vec{v}_K - \vec{v}_N = \vec{v}_k + (-\vec{v}_N) \text{ along the line}$$

$$KN = 0 + [-(-v)] = v$$

Time taken for K and N to meet will be d/v

3. $x = 3 \tan \beta$



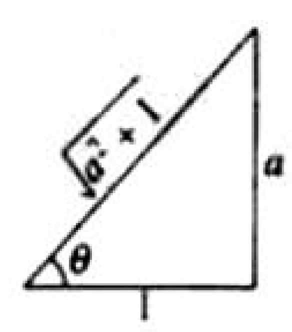
$$\Rightarrow \quad v = \frac{dx}{dt} = 3 \sec^2\beta \, (d\beta/dt)$$

$$\Rightarrow v_{(\beta=45°)} = 3(\sec^2 45°) \times 0.1 = 0.6 \text{ m s}^{-1}$$

4. Given that $y = ax - bx^2$

Comparing it with equation of a projectile, we get

$$y = x \tan\theta - \frac{gx^2}{2u \cos^2\theta} \Rightarrow \tan\theta = a \quad \text{(i)}$$



$$\text{and} \quad \frac{g}{2b \cos\theta} = b \quad \text{(ii)}$$

$$u^2 = \frac{g}{2b \cos^2\theta} = \frac{g(a^2+1)}{2b} \quad \text{(iii)}$$

$$\left[\because \cos\theta = \frac{1}{\sqrt{a^2+1}}\right]$$

Maximum height attained, $H = \dfrac{u^2 \sin^2\theta}{2g}$ (iv)

From Eq. (ii), $\dfrac{g}{2b} = u^2 \cos^2\theta$

$$\Rightarrow \quad \frac{g}{2b} = u^2 - u^2 \sin^2\theta$$

$$\Rightarrow \quad u^2 \sin^2\theta = u^2 - \frac{g}{2b}$$

$$\Rightarrow \quad u^2 \sin^2\theta = \frac{g(a^2+1)}{2b} - \frac{g}{2b} = \frac{ga^2}{2b} \quad \text{(v)}$$

From Eqs. (iv) and (v), $H = \dfrac{ga^2}{2b \times 2g} = \dfrac{a^2}{4b}$ and angle of projection with horizontal is $\theta = \tan^{-1}(a)$

Alternatively:

$$\frac{dy}{dx} = a - 2bx = 0 \text{ (For maximum height)}$$

$$\Rightarrow \quad x = \frac{a}{2b}$$

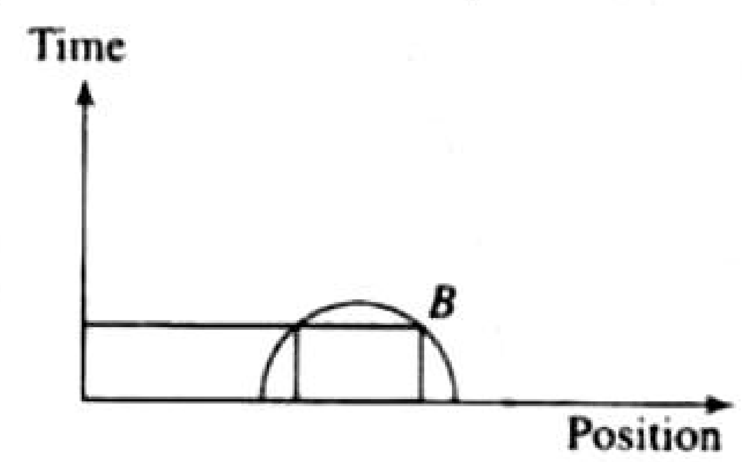
Substituting the value of x in $y = ax - bx^2$ to find maximum height, $H = a\left(\dfrac{a}{2b}\right) - b\left(\dfrac{a^2}{4b^2}\right) = \dfrac{a^2}{2b} - \dfrac{a^2}{2b} = \dfrac{a^2}{4b}$

True or False Type

1. **True.**
   As there is no air resistance, so motion will be independent of mass. Hence, both will reach with same velocity.
2. **True.**
   TE = PE + KE
   TE = Constant
   At P, KE is minimum and PE is maximum.
   Since KE is minimum, velocity is also minimum at P, the topmost point.

Subject Type

1. a. No. As shown, at a given instant of time, the body is at two different positions which is not possible.

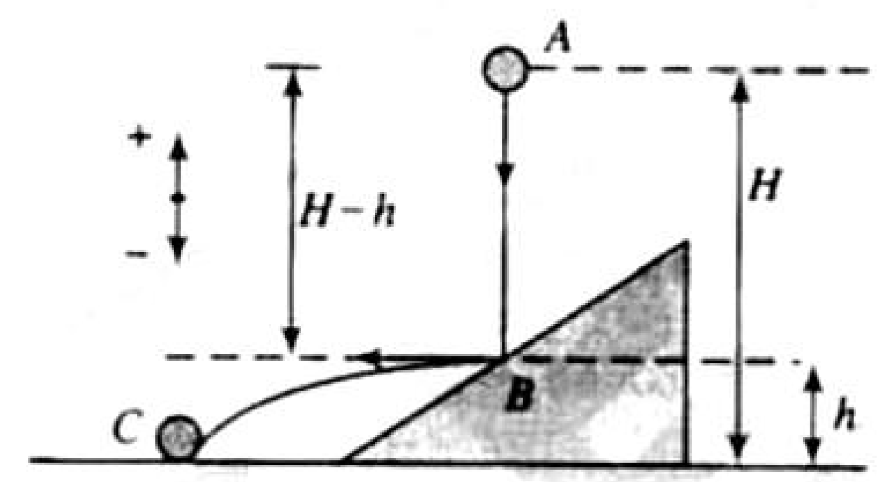


2. For motion from A to B

In vertical direction using $s_y = u_y t + \frac{1}{2} a_y t^2$

$$u_y = 0, s_y = -(H-h) \text{ and } a_y = -g$$

$$-(H-h) = \frac{1}{2}(-g)t_1^2$$



Which gives $t_1 = \left[\dfrac{2(H-h)}{g}\right]^{1/2}$ (i)

For motion from B to C, the body starts moving in horizontal direction

$$u_y = 0, s_y = -h \text{ and } a_y = -g$$

$$-h = -\frac{1}{2}at_2^2 \Rightarrow t_2 = \sqrt{\frac{2h}{g}} \quad \text{(ii)}$$

Total time of fall $T = t_1 + t_2$

$$T = \left[\frac{2(H-h)}{g}\right]^{1/2} + \left[\frac{2h}{g}\right]^{1/2}$$

For maximum time of flight $\frac{dT}{dh} = 0$

$$\Rightarrow \quad \frac{d}{dh}\left[\frac{2(H-h)}{g}\right]^{1/2} + \frac{d}{dh}\left[\frac{2h}{g}\right]^{1/2} = 0$$

$$\Rightarrow \quad \frac{1}{2}\left[\frac{2(H-h)}{g}\right]^{-1/2} \times \left(\frac{-2}{g}\right) + \frac{1}{2}\left[\frac{2h}{g}\right]^{-1/2}\left(\frac{2}{g}\right) = 0$$

$$\Rightarrow \quad \left[\frac{2(H-h)}{g}\right]^{-1/2} = \left[\frac{2h}{g}\right]^{-1/2}$$

$$\Rightarrow \quad \frac{2(H-h)}{g} = \frac{2h}{g}$$

$$\Rightarrow \quad H - h = h \Rightarrow \frac{h}{H} = \frac{1}{2}$$

3. (i) Let collision occurs at time t

For horizontal motion of m:

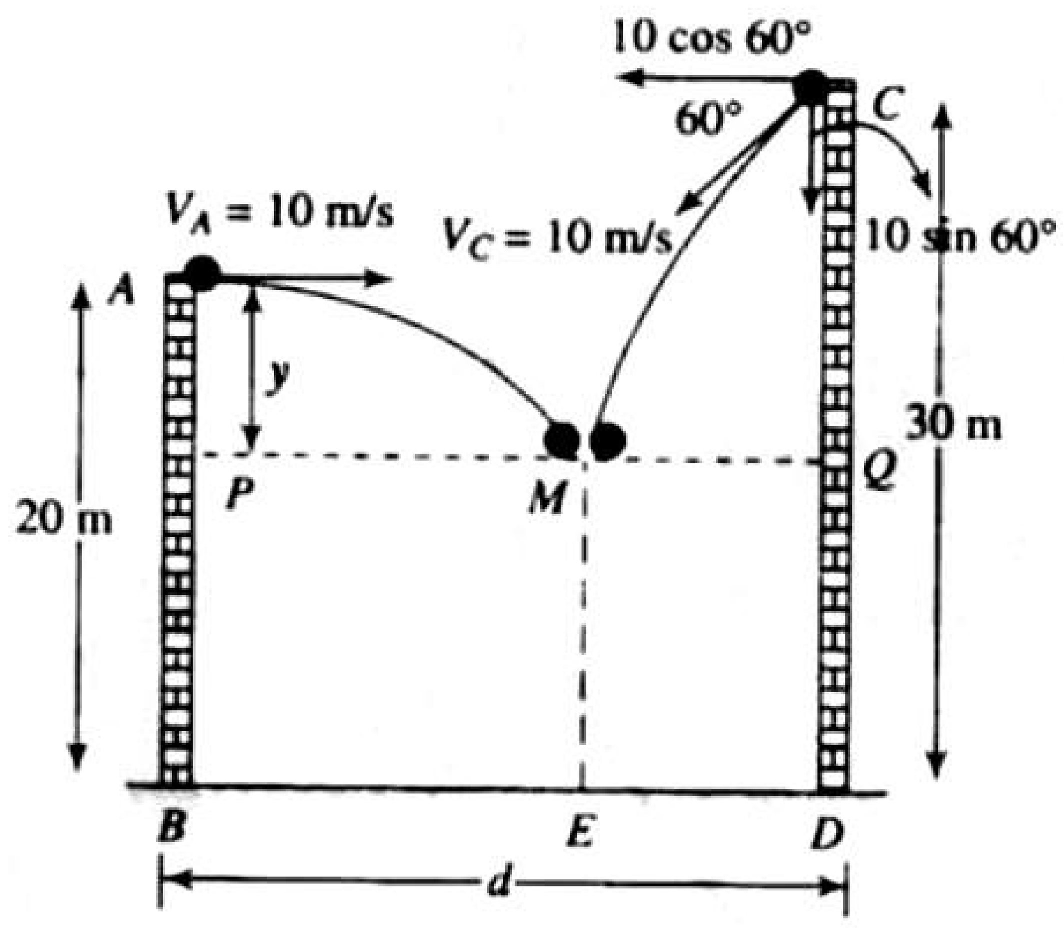
The displacement of A, $PM = 10\,t$ (i)

For vertical motion of m

$u_y = 0$, $s_y = y$ and $a_y = g$

Using $s_y = u_y t + \frac{1}{2}a_y t^2$

$y = \frac{1}{2}gt^2$ (ii)



and $v_y = u + at = gt$ (iii)

For mass 2m thrown from C

$u_x = 10 \cos 60° = 10 \times \frac{1}{2} = 5$ m/s

$u_y = 10 \sin 60° = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}$ m/s

For horizontal motion $QM = [10 \cos 60°]t$

$QM = 5t$ (iv)

For vertical motion

$v_y = 5\sqrt{3} + gt$ (v)

$s_y = u_y t + \frac{1}{2}a_y t^2$

$\therefore \quad s_y = y + 10 \Rightarrow y + 10 = 5\sqrt{3}t + \frac{1}{2}gt^2$ (vi)

From (ii) and (vi)

$$\frac{1}{2}gt^2 + 10 = 5\sqrt{3}t + \frac{1}{2}gt^2 \Rightarrow t = \frac{2}{\sqrt{3}}\text{ sec}$$

$$\therefore \quad BD = PM + MQ = 10t + 5t = 15t = 15 \times \frac{2}{\sqrt{3}}$$

$$= 10\sqrt{3} = 17.32 \text{ m}$$

(ii) Applying conservation of linear momentum (during collision of the masses at M) in the horizontal direction

$m \times 10 - 2\,m\,10 \cos 60° = 3\,m \times v_x$

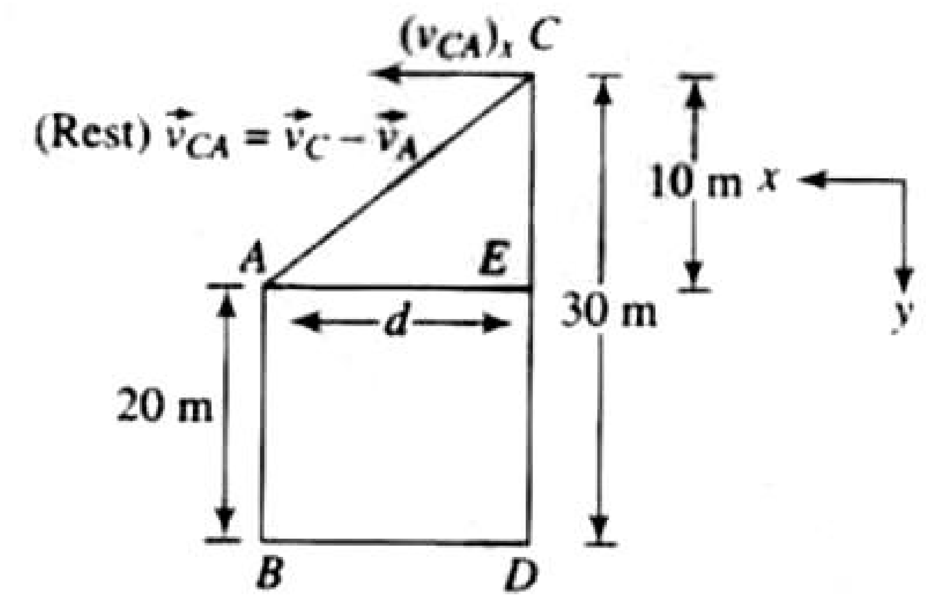
$\Rightarrow \quad 10\,m - 10\,m = 3\,m \times v_x \Rightarrow v_x = 0$

Since the horizontal momentum comes out to be zero, the combination of masses will drop vertically downwards and fall at E.

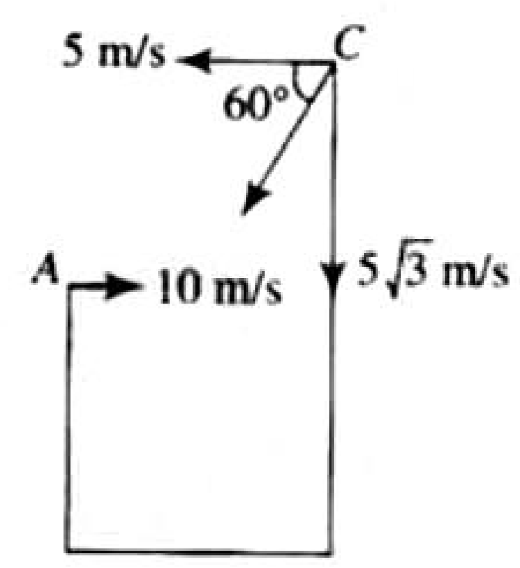
$$BE = PM = 10t = 10 \times \frac{2}{\sqrt{3}} = 11.547 \text{ m}$$

Method 2:

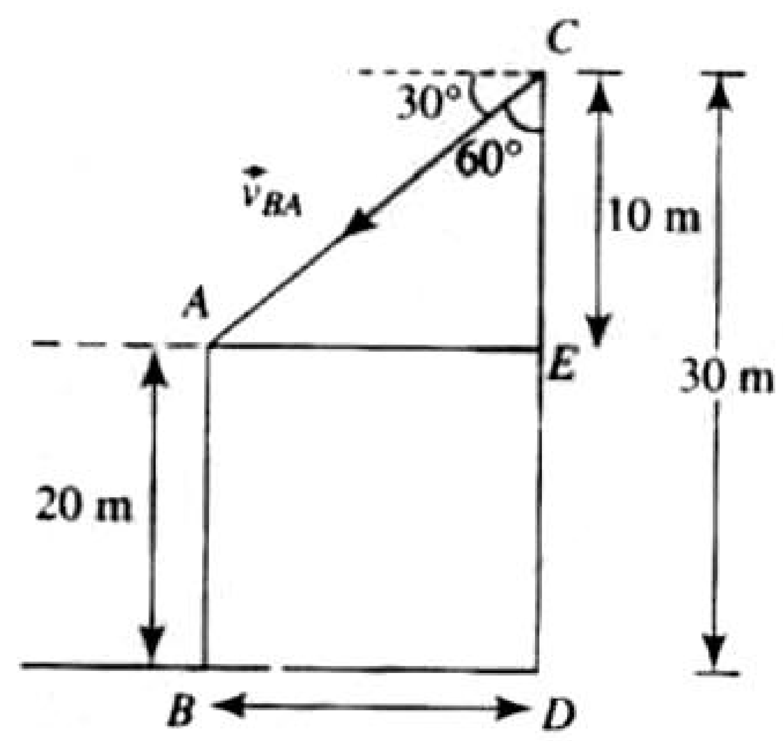
Acceleration of A and C both is 10 m/s² downwards.



Therefore, relative acceleration between them is zero, i.e., the relative motion between them will be a straight line.



Now assuming A to be at rest, the condition of collision will be that $\vec{v}_{CA} = \vec{v}_C - \vec{v}_A$ = relative velocity of C w.r.t A should be along CA.



$\vec{v}_A = 10\hat{i}$

$\vec{v}_B = -5\hat{i} - 5\sqrt{3}\,\hat{j}$

$\vec{v}_{BA} = -5\hat{i} - 5\sqrt{3}\,\hat{j} - 10\hat{i}$

$\therefore \quad \vec{v}_{BA} = -15\hat{i} - 5\sqrt{3}\,\hat{j}$

$\therefore \quad \tan 60° = \dfrac{d}{10}$

$\therefore \quad d = 10\sqrt{3}$ m

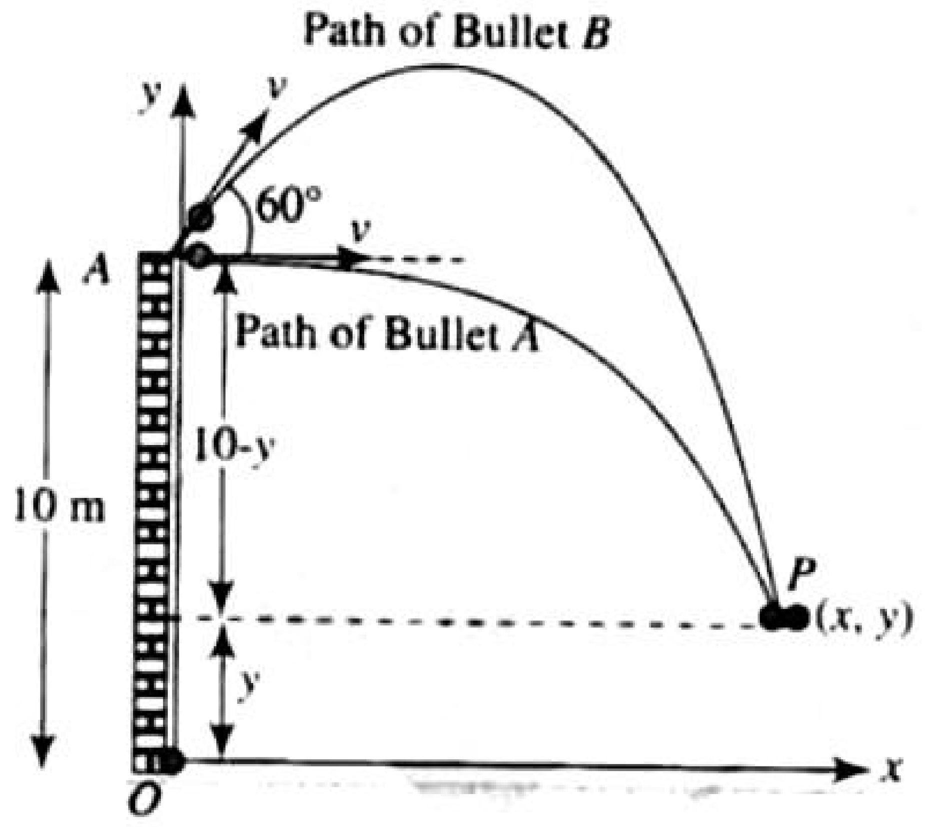
4. Let both bullets collide at P. For bullet A, let t be the time taken by bullet A to reach P.

Motion in vertical direction:

$u_y = 0,\ s_y = 10 - y$ and $a_y = 10$ m/s^2

Using $s_y = u_y t + \dfrac{1}{2}a_y t^2$

$$10 - y = 5t^2 \quad \text{(i)}$$



Now considering horizontal direction:

$$x = 5\sqrt{3}t \quad \text{(ii)}$$

Now consider the motion of bullet B.

Let $(t + t')$ to be the time taken by bullet B to reach P.

In vertical direction

$u_y = +5\sqrt{3}\sin 60° = +7.5$ m/s

$a_y = -10$ m/s^2, $s_y = -(10 - y) = y - 10$

Using $s_y = u_y t + \dfrac{1}{2}a_y t^2$

$$y - 10 = 7.5(t + t') - 5(t + t')^2 \quad \text{(iii)}$$

In horizontal direction

$x = (5\sqrt{3}\cos 60°)(t + t')$

$$\Rightarrow \quad 5\sqrt{3}t + 5\sqrt{3}t' = 2x \quad \text{(iv)}$$

Substituting the value of x from (ii) in (iv) we get

$5\sqrt{3}t + 5\sqrt{3}t' = 10\sqrt{3}t \quad \Rightarrow t = t'$

Putting $t = t'$ in equation (iii)

$$y - 10 = 15 - 20t^2 \quad \text{(v)}$$

Adding (i) and (v)

$0 - 15t - 15t^2 \Rightarrow t = 1$ sec

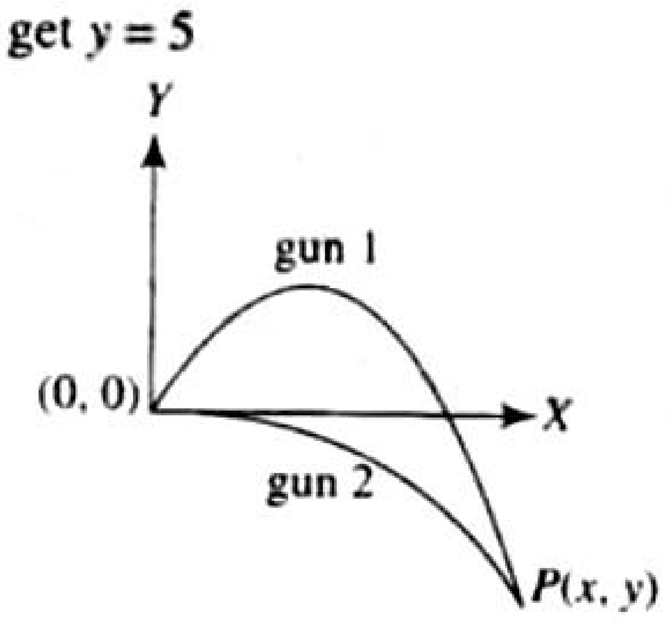
(ii) Putting $t = 1$ in equation (ii) we get $x = 5\sqrt{3}$

Putting $t = 1$ in equation (i), we get $y = 5$

Therefore the coordinates of point P are $(5\sqrt{3}$ m, 5 m$)$

Method 2:

We take point of firing as origin and x and y-axis as shown in the figure. Equation of trajectory of a projectile is



$$y = x\tan\theta - \frac{gx^2}{2v_i^2\cos^2\theta}$$

For gun 1, $\theta = 60°$

$$y = x\tan 60° - \frac{gx^2}{2v_i^2\cos^2 60°}$$

$$= x\sqrt{3} - \frac{2gx^2}{v_i^2}$$

For gun 2, $\theta = 0°$

$$y = \frac{-gx^2}{2v_i^2}$$

Two shots collide at point P; therefore their coordinates must be the same.

$$\frac{-gx^2}{2v_i^2} = x\sqrt{3} - \frac{2gx^2}{2v_i^2}$$

$$x\sqrt{3} = \frac{2gx^2}{v_i^2} - \frac{gx^2}{2v_i^2} = \frac{3gx^2}{2v_i^2}$$

$$x = 0 \text{ and } x = \frac{2v_i^2}{\sqrt{3}g} = \frac{2(5\sqrt{3})^2}{\sqrt{3}(10)} = 5\sqrt{3}\text{ m}$$

$$y = \frac{-gx^2}{2v_i^2} = \frac{-10(5\sqrt{3})^2}{2(5\sqrt{3})^2} = -5\text{ m}$$

If origin is assigned at ground the coordinates of point P will be $(5\sqrt{3}$ m, 5 m$)$.

Now we consider x-component of displacement for both the shots.

Gun 1: $x = 5\sqrt{3}\text{ m} = v_i t = (5\sqrt{3}\text{ m/s})t$

or $\quad t_1 = 1$ s

Gun 2: $x = 5\sqrt{3}\text{ m} = v_i\cos 60°\, t_2 = \dfrac{5\sqrt{3}}{2}t_2$

or $\quad t_2 = 2$ s

Time interval between two shots is $\Delta t = t_2 - t_1 = 1$ s

5. i. Let u is the velocity of the particle with respect to the box. Considering the motion parallel and perpendicular to inclined plane.

u_x is the relative velocity of particle with respect to the box in x-direction.

u_y is the velocity with respect to the box in y-direction.

y-direction motion (w.r.t. box)

$u_y = +u\sin\alpha$

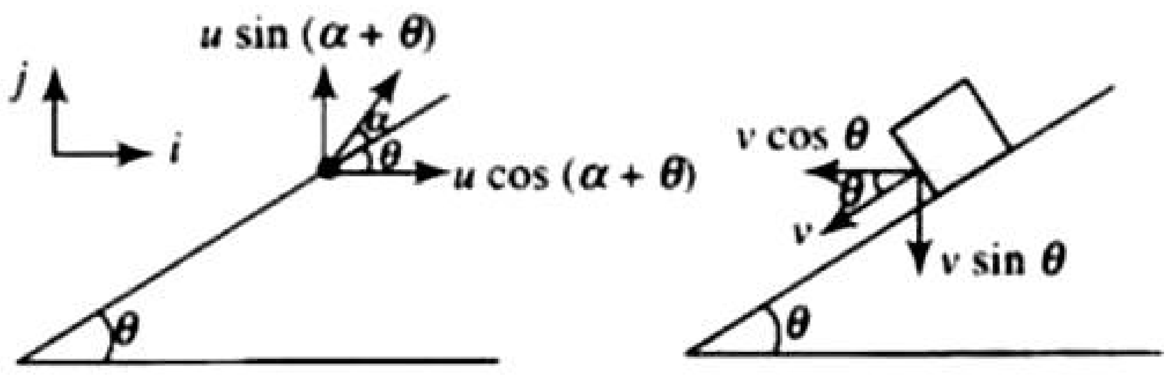
$a_y = -g\cos\theta$

$s_Y = 0$ (motion is taken till the time the particle comes back to the box)

$$s_y = u_y t + \frac{1}{2}a_y t^2 \Rightarrow 0 = (u\sin\alpha)t - \frac{1}{2}g\cos\theta \times t^2$$

$$\Rightarrow \quad t = 0 \text{ or } t = \frac{2u\sin\alpha}{g\cos\theta}$$

ii. Horizontal displacement of particle with respect to ground is zero. It means that initial velocity with respect to ground is only vertical, or there is no horizontal component of the absolute velocity of the particle.



Let v be the velocity of the block down the plane.
Velocity of particle

$$= u\cos(\alpha+\theta)\hat{i} + u\sin(\alpha+\theta)\hat{j}$$

Velocity of block $= -v\cos\theta\hat{i} - v\sin\theta\hat{j}$

$\therefore$ velocity of particle with respect to ground

$$= \{u\cos(\alpha+\theta) - v\cos\theta\}\hat{i} + \{u\sin(\alpha+\theta) - v\sin\theta\}\hat{j}$$

Now, as we said earlier that horizontal component of absolute velocity should be zero.
Therefore, $u\cos(\alpha+\theta) - v\cos\theta = 0$

or $\quad v = \dfrac{u\cos(\alpha+\theta)}{\cos\theta}$ (down the plane)

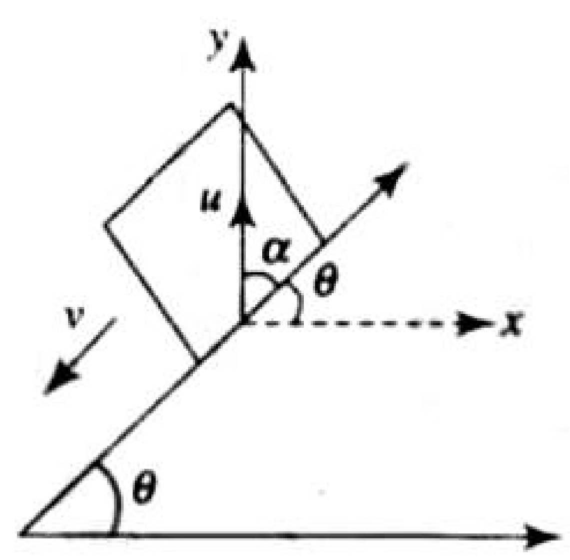
Method 2:
The above condition can be meet if the box covers exactly the same distance as the range of particle,

$$\left(\frac{u^2\sin 2\alpha}{g\cos\theta}\right) = v\left(\frac{2u\sin\alpha}{g\cos\theta}\right) + \frac{1}{2}g\sin\theta\left(\frac{2u\sin\alpha}{g\cos\theta}\right)^2$$

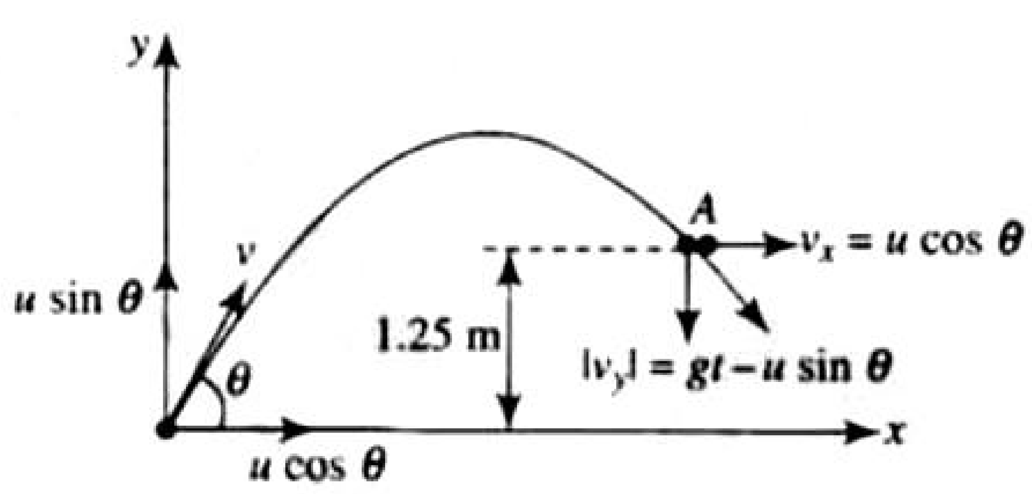
or $\quad u\cos\alpha = v + \dfrac{u\sin\theta\sin\alpha}{\cos\theta}$

or $\quad v = u\left(\dfrac{\cos\alpha\cos\theta - \sin\alpha\cos\theta}{\cos\theta}\right)$

$$= \frac{u\cos(\alpha+\theta)}{\cos\theta}$$



6. Let after time 't' the stone hits the object and θ be the angle which the velocity vector $\vec{u}$ makes with horizontal.



From question, we have following three informations

(i) Vertical displacement of stone is 1.25 m.

Using $s_y = u_y t + \dfrac{1}{2}a_y t^2$

Therefore $1.25 = (u\sin\theta)t - \dfrac{1}{2}gt^2$

or $\quad (u\sin\theta)t = 1.25 + 5t^2$ (i)

(ii) Horizontal displacement of stone
= 3 + displacement of object A.

Therefore $(u\cos\theta)t = 3 + \dfrac{1}{2}at^2$

We have $a = 1.5$ m/s^2
hence $(u\cos\theta)\,t = 3 + 0.75\,t^2$ (ii)

(iii) Horizontal component of velocity (of stone) = vertical component (because velocity vector is inclined) at 45° with horizontal).
Therefore $(u\cos\theta) = gt - (u\sin\theta)$ (iii)
(The right hand side is written $gt - u\sin\theta$ because the stone is in its downward motion. Therefore, $gt > u\sin\theta$. In upward motion $u\sin\theta > gt$). Multiplying equation (iii) with t we can write,

$$(u\cos\theta)t + (u\sin\theta)t = 10t^2 \quad \text{(iv)}$$

Now, doing (iv) – [(ii) + (i)]
It gives $4.25\,t^2 - 4.25 = 0$ or $t = 1$ s
Substituting $t = 1$ s in (i) and (ii) we get,
$u\sin\theta = 6.25$ m/s or $u_y = 6.25$ m/s
and $\quad u\cos\theta = 3.75$ m/s or $u_x = 3.75$ m/s

Now we can write $\vec{u} = u_x\hat{i} + u_y\hat{j}$

or $\quad \vec{u} = (3.75\hat{i} + 6.25)\hat{j}$ m/s

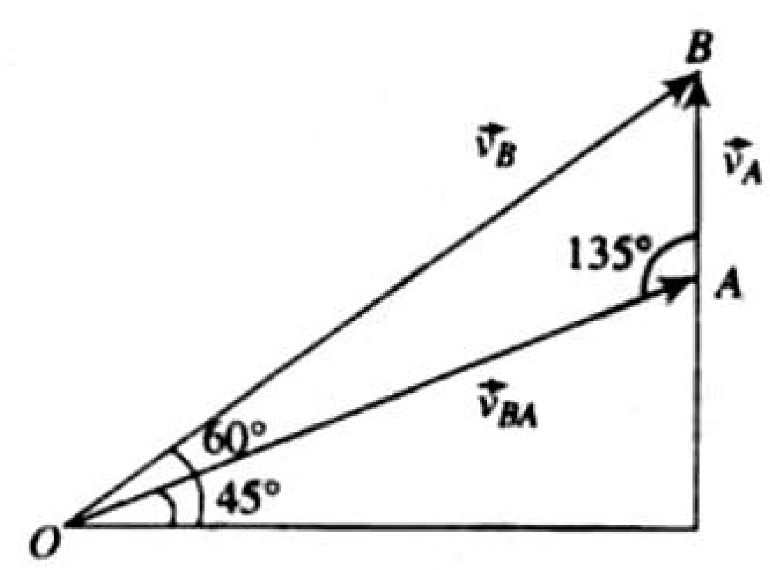
7. **Method 1:**

a. Since the ball hits the trolley, relative to trolley. The velocity of ball should be directed towards the trolley. Hence, in the frame of trolley, the ball will appear to be moving towards OA, or in the frame of trolley, ball's velocity will make an angle of 45°.

b. $\phi = \dfrac{4\theta}{3} = \dfrac{4\times 45°}{3} = 60°$

Using sine rule $\dfrac{V_B}{\sin 135°} = \dfrac{V_T}{\sin 15°}$

$\Rightarrow V_B = 2$ ms^{-1}



Method 2:

a. Let A stands for trolley and B for ball.
Relative velocity of B with respect to $A(\vec{v}_{BA})$ should be along OA for the ball to hit the trolley.
Hence $\vec{v}_{BA}$ will make an angle of 45° with positive x-axis.

b. $\tan\theta = \dfrac{v_{BAy}}{v_{BAx}} = \tan 45°$

or $\quad v_{BA_y} = v_{BA_x}$
Further $V_{BA_y} = V_{B_y} - V_{A_y}$
or $\quad V_{BA_x} = V_{B_x} - 0$
$v_{BA_y} = v_{B_y} - (\sqrt{3} - 1)$

$$\tan\theta = \frac{v_{By}}{v_{Bx}}$$

or $\quad v_{By} = v_{Bx}\tan\phi \qquad$ (iv)

From equations (i), (ii), (iii) and (iv), we get

$$v_{Bx} = \frac{(\sqrt{3}-1)}{\tan\phi - 1} \text{ and } v_{By} = \frac{(\sqrt{3}-1)}{\tan\theta - 1}\cdot\tan\phi$$

$$\phi = \frac{4\theta}{3} = \frac{4}{3}(45^\circ)$$

Speed of ball w.r.t surface $v_B = \sqrt{v_{Bx}^2 + v_{By}^2}$

$$= \frac{\sqrt{3}-1}{\tan\phi - 1}\sec\phi$$

Substituting $\phi = 60^\circ$, we get $v_B = 2$ m/s